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ERRORS IN TEMPERATURE MEASUREMENT ON THE SURFACE OF A SOLID BODY USING A THERMOCOUPLE WHEN HEATING AND COOLING FOLLOW AN ARBITRARY LAW

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ERRORS IN TEMPERATURE MEASUREMENT ON THE SURFACE OF A SOLID BODY USING A THERMOCOUPLE WHEN HEATING AND COOLING FOLLOW AN ARBITRARY LAW

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ABSTRACT 29736

Equations are derived for determining errors in temperature measurement on the surface of a semi-infinite solid body by means of a semi-artificial thermocouple, when heating and cooling take place from some stationary temperature state in accordance with an arbitrary law. Author

Let us determine the measurement error which occurs when temperature is /60*
measured with a semi-artificial thermocouple on the surface of a semi-infinite solid body, an error produced by the heat flow along the electrode of the thermocouple. The thermal flux along the electrode produces a temperature field which will be superimposed on the temperature field of a semi-infinite body, and which will tend to distort it. This distortion of the temperature field determines the error in the measurement of temperature. The thermal flux along the electrode depends on the parameters of the electrode and on the temperature at the contact area between the electrode and the body (which is

*Numbers given in the margin indicate the pagination in the original foreign text.

measured with a thermocouple); therefore, the distorting temperature field and the measurement error can be determined from the measured temperature independently of the true temperature field of the body. The following assumptions are made in the solution of this problem: the electrode is semi-infinite, the coefficient of heat exchange on the surface of the electrode a is constant and is the same for the entire surface, the temperature gradient along the cross section of the electrode is absent, the thermal flux is constant along the radius of the electrode R_e and the surface of the body is thermally insulated (this has practically no effect on the measurement error). We shall assume that the temperature of the surrounding medium is equal to zero. At the initial instant of time ($\tau = 0$) the electrode and the body will have a stationary temperature distribution different from 0. The temperature field of the electrode and the distorting temperature field in the body at the initial instant of time are described by the following equations (ref. 1)

(1)

$$t_2 = -\frac{q_0 R_e}{\lambda_T} \int_0^\infty \frac{1}{\gamma} \exp(-\gamma |x|) J_1(\gamma R_e) J_0(\gamma r)\, d\gamma, \quad x < 0, \tag{2}$$

where $M_e = \frac{2\alpha_0}{\lambda_e R_e}$, $q_0 = \lambda_e t_{k_0} \sqrt{m_0}$ (the origin of this particular coordinate system is at the center of the area of tangency between the electrode and the body). The negative sign in equation (2) shows that the distorting temperature field decreases the undistorted temperature field. After this the body will be heated or cooled, and the temperature on the contact area will vary according to an arbitrary law $t_k = t_k + (\tau)$, where (τ) will have a plus sign during heating and a minus sign during cooling, while at the initial instant of time it will be equal to 0. The thermal flux on the contact area $q(\tau)$ may be determined by solving the equation of heat conductivity for the electrode /61

$$\frac{\partial^2 t_1}{\partial x^2} - m t_1 = \frac{1}{a_3}\frac{\partial t_1}{\partial \tau}, \quad x > 0. \tag{3}$$

where m = $2\alpha/\lambda_e R_e$

To solve equation (3) we use the Laplace transformation with the initial condition (1)

$$\frac{\partial^2 \bar{t}_1}{\partial x^2} - \left(m + \frac{s}{a_3}\right)\bar{t}_1 = - t_{K_0}\frac{1}{a_3}\exp(-x\sqrt{m_0}). \tag{4}$$

The solution of this equation, which satisfies the condition $\bar{t}_1|_{x=\infty} < \infty$ and the transformed boundary conditi $\bar{t}_1|_{x=0} = \frac{t_{K_0}}{s} + F(s),$ is

$$\bar{t}_1 = \left[F(s) + \frac{t_{K_0}}{s} - \frac{t_{K_0}}{a_3(m-m_0)+s}\right]\exp\left(-x\sqrt{m+\frac{s}{a_3}}\right) + \frac{t_{K_0}\exp(-x\sqrt{m_0})}{a_3(m-m_0)+s}. \tag{5}$$

The image of the thermal flux on the contact area is given by expression

$t_1 = t_{K_0}\exp(-x\sqrt{m_0}), \quad x > 0;$

$$Q(s) = -\lambda_3\frac{\partial \bar{t}_1}{\partial x}\Big|_{x=0} = \frac{\lambda_3}{\sqrt{a_3}}F(s)\sqrt{ma_3+s} + t_{K_0}\lambda_3\left[\frac{\sqrt{a_3}(m-m_0)\sqrt{ma_3+s}}{s[a_3(m-m_0)+s]} + \frac{\sqrt{m_0}}{a_3(m-m_0)+s}\right]. \tag{6}$$

The distorting temperature field produced by the thermal flux can be obtained by solving the equations of heat conductivity for the body, using a cylindrical system of coordinates

$$\frac{\partial^2 t_2}{\partial r^2} + \frac{1}{r}\frac{\partial t_2}{\partial r} + \frac{\partial^2 t_2}{\partial x^2} = \frac{1}{a_T}\frac{\partial t_2}{\partial \tau}, \quad x < 0. \tag{7}$$

To solve equation (7) we use the initial condition (2) and the boundary condition.

$$\left[\frac{\partial t_2}{\partial x}\right]_{x=0} = -\frac{q(\tau)}{\lambda_T}\Big|_{r \leq R_3}, \quad |t_2|_{x\to-\infty} < \infty. \tag{8}$$

and apply the integral transformation of Heinkel and Laplace. After this the equation takes the form

$$\frac{\partial^2 \bar{t}_2}{\partial x^2} - \left(\gamma^2 + \frac{s}{a_\tau}\right)\bar{t}_2 = \frac{1}{a_\tau}\,\frac{q_0 R_2 J_1(\gamma R_2)}{\lambda_\tau \gamma^2}\exp(-\gamma|x|). \tag{9}$$

while the boundary conditions (8) is given by expressions /62

$$\left[\frac{\partial \bar{t}_2}{\partial x}\right]_{x=0} = -\frac{Q(s) R_2 J_1(\gamma R_2)}{\lambda_\tau \gamma}, \quad |\bar{t}_2|_{x\to\infty} < \infty. \tag{10}$$

The solution of equation (9), taking into account boundary conditions (10) and equation (6) has the form

$$\begin{aligned}
\bar{t}_2 = &-\frac{\lambda_2\sqrt{a_\tau}R_2J_1(\gamma R_2)}{\lambda_\tau\sqrt{a_2}\,\gamma}F(s)\frac{\sqrt{ma_2+s}}{\sqrt{a_\tau\gamma^2+s}}\,\frac{s}{s}\exp\left(-\frac{|x|}{\sqrt{a_\tau}}\sqrt{a_\tau\gamma^2+s}\right)-\\
&-t_{K_0}\frac{\lambda_2\sqrt{a_2}R_2J_1(\gamma R_2)}{\lambda_\tau\gamma}\left[\frac{\sqrt{a_2}(m-m_0)\sqrt{ma_2+s}}{s\,[a_2(m-m_0)+s]\,\sqrt{a_\tau\gamma^2+s}}\times\right.\\
&\times\exp\left(-\frac{|x|}{\sqrt{a_\tau}}\sqrt{a_\tau\gamma^2+s}\right)+\left.\frac{\sqrt{m_0}\exp(-|x|\sqrt{a_\tau\gamma^2+s}/\sqrt{a_\tau})}{[a_2(m-m_0)+s]\sqrt{a_\tau\gamma^2+s}}\right]+\\
&+t_{K_0}\frac{\lambda_2\sqrt{m_0}\sqrt{a_\tau}R_2J_1(\gamma R_2)}{\lambda_\tau\gamma s\sqrt{a_\tau\gamma^2+s}}\exp\left(-\frac{|x|}{\sqrt{a_\tau}}\sqrt{a_\tau\gamma^2+s}\right)-\\
&-t_{K_2}\frac{\lambda_2\sqrt{m_0}R_2J_1(\gamma R_2)}{\lambda_\tau\gamma^2 s}\exp(-\gamma|x|).
\end{aligned} \tag{11}$$

Then, by applying the inverse Laplace and Heinkel transformation we find the distorting temperature field. In the inverse Laplace transformation we utilize the following equations, which establish a relationship between the image and the original (ref. 2)

$$\begin{aligned}
&sF(s) = \{f'(\tau)\},\\
&\frac{\sqrt{ma_2+s}}{s} = \left\{\frac{1}{\sqrt{\pi\tau}}\exp(-ma_2\tau)+\sqrt{ma_2}\,\mathrm{erf}\sqrt{ma_2\tau}\right\},\\
&\frac{\exp(-|x|\sqrt{a_\tau\gamma^2+s}/\sqrt{a_\tau})}{\sqrt{a_\tau\gamma^2+s}} = \left\{\frac{1}{\sqrt{\pi\tau}}\exp\left(-\frac{x^2}{4a_\tau\tau}-a_\tau\gamma^2\tau\right)\right\}.
\end{aligned} \tag{12}$$

then we obtain

$$t_2 = -\frac{\lambda_{\text{з}}\sqrt{a_{\text{т}}}R_{\text{з}}}{\lambda_{\text{т}}\sqrt{a_{\text{з}}}}\int_0^\infty\left\{\int_0^\tau\left[\int_0^{\tau-t} f'(\tau-t-s)\frac{1}{\sqrt{\pi s}}\times\right.\right.$$
$$\left.\times\exp\left(-\frac{x^2}{4a_{\text{т}}s}-a_{\text{т}}\gamma^2 s\right)ds\right]\left(\frac{1}{\sqrt{\pi t}}\exp(-ma_{\text{з}}t)+\right.$$
$$\left.\left.+\sqrt{ma_{\text{з}}}\operatorname{erf}\sqrt{ma_{\text{з}}t}\right)dt\right\}J_1(\gamma R_{\text{з}})J_0(\gamma r)d\gamma-$$
$$-t_{K_0}\frac{\lambda_{\text{з}}\sqrt{a_{\text{т}}}R_{\text{з}}}{\lambda_{\text{т}}}\sqrt{a_{\text{з}}}(m-m_0)\int_0^\infty\left\{\int_0^\tau\left[\int_0^{\tau-t}\exp\left[-a_{\text{з}}(m-m_0)(\tau-t-s)\right]\times\right.\right.$$
$$\left.\times\frac{1}{\sqrt{\pi s}}\exp\left(-\frac{x^2}{4a_{\text{т}}s}-a_{\text{т}}\gamma^2 s\right)ds\right]\left(\frac{1}{\sqrt{\pi t}}\exp(-ma_{\text{з}}t)+\right.$$
$$\left.\left.+\sqrt{ma_{\text{з}}}\operatorname{erf}\sqrt{ma_{\text{з}}t}\right)dt\right\}J_1(\gamma R_{\text{з}})J_0(\gamma r)d\gamma-$$
$$-t_{K_0}\frac{\lambda_{\text{з}}\sqrt{a_{\text{т}}}R_{\text{з}}\sqrt{m_0}}{\lambda_{\text{т}}}\int_0^\infty\left[\int_0^\tau\exp\left[-a_{\text{з}}(m-m_0)(\tau-t)\right]\times\right.$$
$$\left.\times\frac{1}{\sqrt{\pi t}}\exp\left(-\frac{x^2}{4a_{\text{т}}t}-a_{\text{т}}\gamma^2 t\right)dt\right]J_1(\gamma R_{\text{з}})J_0(\gamma r)d\gamma+$$
$$+t_{K_0}\frac{\lambda_{\text{з}}\sqrt{a_{\text{т}}}\sqrt{m_0}R_{\text{з}}}{\lambda_{\text{т}}}\int_0^\infty\left[\int_0^\tau\frac{1}{\sqrt{\pi t}}\exp\left(-\frac{x^2}{4a_{\text{т}}t}-a_{\text{т}}\gamma^2 t\right)dt\right]\times$$
$$\times J_1(\gamma R_{\text{з}})J_0(\gamma r)d\gamma-t_{K_0}\frac{\lambda_{\text{з}}\sqrt{m_0}R_{\text{з}}}{\lambda_{\text{т}}}\int_0^\infty\frac{1}{\gamma}J_1(\gamma R_{\text{з}})J_0(\gamma r)\times$$
$$\times\exp(-\gamma|x|)d\gamma. \qquad (13)$$

Replacing the integration variables and introducing criteria of similarity $\mathrm{Fo}=a_{\text{з}}\tau/R_{\text{з}}^2$, $\mathrm{Bi}=\alpha R_{\text{з}}/\lambda_{\text{з}}$, $\mathrm{Bi}_0=\alpha_0 R_{\text{з}}/\lambda_{\text{з}}$, $k_l=|x|/R_{\text{з}}$, $k_r=r/R_{\text{з}}$, $k_\lambda=\lambda_{\text{з}}/\lambda_{\text{т}}$, $k_a=a_{\text{з}}/a_{\text{т}}$, we obtain the following expression for the distorting temperature field /63

$$t_2=-\frac{2\sqrt{2}k_\lambda\tau}{\pi\sqrt{\mathrm{Bi}\,\mathrm{Fo}}}\int_0^\infty\left\{\int_0^{\sqrt{2\mathrm{BiFo}}}\left[\int_0^{z\sqrt{\frac{\mathrm{Fo}}{k_a}\left(1-\frac{u^2}{2\mathrm{BiFo}}\right)}} f'\left[\tau\left(1-\frac{u^2}{2\mathrm{BiFo}}-\right.\right.\right.\right.$$
$$\left.\left.\left.-\frac{v^2k_a}{z^2\mathrm{Fo}}\right)\right]\exp\left(-\frac{z^2}{4v^2}k_l^2-v^2\right)dv\right]\left[\exp(-u^2)+\sqrt{\pi}\,u\operatorname{erf}u\right]du\right\}\times$$
$$\times\frac{1}{z}J_1(z)J_0(k_rz)dz-t_{K_0}\frac{4\sqrt{2}k_\lambda(\mathrm{Bi}-\mathrm{Bi}_0)}{\pi\sqrt{\mathrm{Bi}}}\times$$
$$\times\exp\left[-2\mathrm{Fo}(\mathrm{Bi}-\mathrm{Bi}_0)\right]\int_0^\infty\left\{\int_0^{\sqrt{2\mathrm{BiFo}}}\left[\int_0^{z\sqrt{\frac{\mathrm{Fo}}{k_a}\left(1-\frac{u^2}{2\mathrm{BiFo}}\right)}}\exp\left[\frac{2v^2k_a}{z^2}(\mathrm{Bi}-\mathrm{Bi}_0)\right]\times\right.\right.$$

$$
\begin{gathered}
\times \exp\left(-\frac{z^2}{4v^2}k_l^2 - v^2\right)dv\Big][\exp(-u^2) + \sqrt{\pi}\,u\,\mathrm{erf}\,u] \times \\
\times \exp\left[u^2\left(1-\frac{\mathrm{Bi}_0}{\mathrm{Bi}}\right)\right]du\Big\}\frac{1}{z}J_1(z)J_0(k_r z)\,dz - \\
- t_{K_0}\frac{2\sqrt{2}\sqrt{\mathrm{Bi}_0}}{\sqrt{\pi}}k_\lambda \exp[-2\mathrm{Fo}(\mathrm{Bi}-\mathrm{Bi}_0)]\int_0^\infty\Big\{\int_0^{z\sqrt{\frac{\mathrm{Fo}}{k_a}}}\exp\left[\frac{2v^2k_a}{z^2}(\mathrm{Bi}-\mathrm{Bi}_0)\right]\times \\
\times \exp\left(-\frac{z^2}{4v^2}k_l^2 - v^2\right)dv\Big\}\frac{1}{z}J_1(z)J_0(k_r z)\,dz - \\
- t_{K_0}\sqrt{2}\,k_\lambda\sqrt{\mathrm{Bi}_0}\int_0^\infty \frac{1}{z}J_1(z)J_0(k_r z)\exp(-k_l z)\,dz + \\
+ t_{K_0}\frac{2\sqrt{2}\sqrt{\mathrm{Bi}_0}\,k_\lambda}{\sqrt{\pi}}\int_0^\infty\Big[\int_0^{z\sqrt{\frac{\mathrm{Fo}}{k_a}}}\exp\left(-\frac{z^2}{4v^2}k_l^2 - v^2\right)dv\Big]\times \\
\times \frac{1}{z}J_1(z)J_0(k_r z)\,dz.
\end{gathered}
\qquad (14)
$$

The temperature measurement error is determined by the distortion of /64
the temperature field on the contact surface ($x = 0$, $r \leq R_e$)

$$
\begin{gathered}
t_1\Big|_{\substack{x=0\\ r\leq R_e}} = -\frac{2\sqrt{2}\,k_\lambda \tau}{\pi\sqrt{\mathrm{Bi}\,\mathrm{Fo}}}\int_0^\infty\Big\{\int_0^{\sqrt{2\mathrm{BiFo}}}\Big[\int_0^{z\sqrt{\frac{\mathrm{Fo}}{k_a}\left(1-\frac{u^2}{2\mathrm{BiFo}}\right)}} f\Big[\tau\Big(1-\frac{u^2}{2\mathrm{BiFo}} - \\
-\frac{v^2k_a}{z^2\mathrm{Fo}}\Big)\Big]\exp(-v^2)\,dv\Big][\exp(-u^2)+\sqrt{\pi}\,u\,\mathrm{erf}\,u]\,du\Big\}\frac{1}{z}J_1(z)J_0(k_r z)\,dz - \\
- t_{K_0}\frac{4\sqrt{2}\,k_\lambda(\mathrm{Bi}-\mathrm{Bi}_0)}{\pi\sqrt{\mathrm{Bi}}}\times \\
\times \exp[-2\mathrm{Fo}(\mathrm{Bi}-\mathrm{Bi}_0)]\int_0^\infty\Big\{\int_0^{\sqrt{2\mathrm{BiFo}}}\Big[\int_0^{z\sqrt{\frac{\mathrm{Fo}}{k_a}\left(1-\frac{u^2}{2\mathrm{BiFo}}\right)}}\exp\Big\{v^2\Big[\frac{2k_a}{z^2}\times \\
\times(\mathrm{Bi}-\mathrm{Bi}_0)\quad 1\Big]\Big\}dv\Big][\exp(-u^2)+\sqrt{\pi}\,u\,\mathrm{erf}\,u]\exp\left[u^2\left(1-\frac{\mathrm{Bi}_0}{\mathrm{Bi}}\right)\right]du\Big\}\times \\
\times\frac{1}{z}J_1(z)J_0(k_r z)\,dz - t_{K_0}\frac{2\sqrt{2}\sqrt{\mathrm{Bi}_0}}{\sqrt{\pi}}k_\lambda\times \\
\times\exp[\quad 2\mathrm{Fo}(\mathrm{Bi}-\mathrm{Bi}_0)]\int_0^\infty\Big\{\int_0^{z\sqrt{\frac{\mathrm{Fo}}{k_a}}}\exp\Big\{v^2\Big[\frac{2k_a}{z^2}(\mathrm{Bi}-\mathrm{Bi}_0)-1\Big]\Big\}dv\times \\
\times\frac{1}{z}J_1(z)J_0(k_r z)\,dz \quad t_{K_0}\sqrt{2}\,k_\lambda\sqrt{\mathrm{Bi}_0}\int_0^\infty\frac{1}{z}J_1(z)J_0(k_r z)\times \\
\times\Big[1-\mathrm{erf}\Big(z\sqrt{\frac{\mathrm{Fo}}{k_a}}\Big)\Big]dz.
\end{gathered}
\qquad (15)
$$

Thus, the temperature measurement error may be obtained by means of a semi-artificial thermocouple for any instant of time using equation (15), if we have a recorded temperature curve; in this case it is necessary to use the /65 temperature variation curve to construct the variation in its derivative as a function of time. The analysis of equation (15) shows that when the body is heated from some stationary temperature state the measurement error will have a negative sign, i.e., the undistorted temperature will be greater than the measured temperature by the magnitude of this error. When the body is cooled, two cases are possible: (1) the error has a negative sign and (2) the error has a positive sign. In the first case the undistorted temperature is greater, while in the second case it is less than the measured temperature. The error sign during cooling depends on the rate of temperature change. If the initial temperature of the electrode and of the body is equal to zero, i.e., $t_{k_0} = 0$, the measurement error is determined only by the first term of equation (15).

Symbols Used

R --electrode radius; t_{K_0}--temperature on contact surface during stationary temperature state; λ_e, $\lambda_{T\bar{n}}$-coefficients of heat conductivity of electrode and of body; α_e, α_T--coefficients of temperature conductivity of electrode and of body; α_0, α--coefficients of heat exchange on surface of electrode at initial moment of time and during middle of temperature measurement interval; $J_1(\gamma R_e)$, $J_0(\gamma r)$--Bessel functions of first kind of first and zero order; q_0, $q(\tau)$--thermal fluxes on contact surface at initial and variable instants of time; erf u--probability integrals; $F(s)$, $Q(s)$--Laplace transformations of functions $f(\tau)$ and $q(\tau)$.

Summary

The article is concerned with the measurement of temperature on surfaces of solids by means of a semi-artificial thermocouple, involving an error due to the heat transfer along the thermocouple electrode. An equation is derived for determining the distorting temperature field due to heat transfer along the electrode thermocouple, as well as an equation for computing the temperature measurement area when a solid is heated or cooled from a certain stationary temperature in an arbitrary manner. These equations determine the actual undistorted temperature field from the field measured with a thermocouple.

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